2022

Penrith High School
HIGHER SCHOOL CERTIFICATE
**TRIAL EXAMINATION**

# Mathematics Advanced

ALTIORA PETO

**General Instructions**

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- In Questions 11–34 show relevant mathematical reasoning and/or calculations
- Write your NESA ID below, on the Multiple Choice Answer Sheet and the front of Booklets 1 & 2.

**Total marks: 100**

**Section I – 10 marks** (pages 3-8)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

**Section II – 91 marks** (pages 9-32)

- Attempt Questions 11–34
- Allow about 2 hours and 45 minutes for this section

**NESA NUMBER:**

**TEACHER:** ____________________

| Multiple Choice: | Differentiation | Statistics | Algebra & Graphing | Integration | Series | Trigonometry |
|---|---|---|---|---|---|---|
| | /1<br>9 | /2<br>2, 6 | /4<br>3, 4, 7, 8 | /1<br>10 | /1<br>5 | /1<br>1 |

| Differentiation | Statistics | Algebra & Graphing | Integration | Logarithms & Exponentials | Probability | Series | Trigonometry |
|---|---|---|---|---|---|---|---|
| /16 | /10 | /9 | /19 | /8 | /10 | /9 | /20 |
| | | | | | | Total: | /101 |

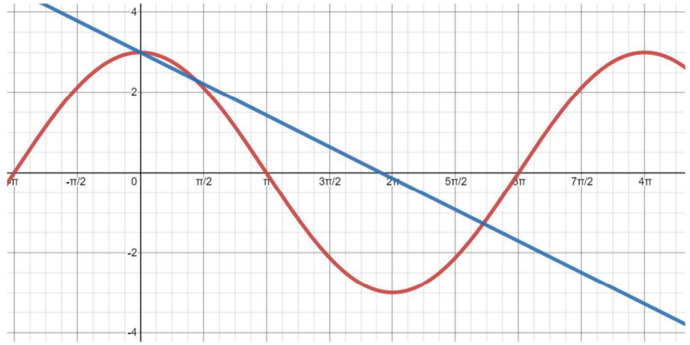
## Section I

**10 marks**
**Attempt questions 1-10**
**Allow about 15 minutes for this section**

Use the multiple-choice answer sheet for Questions 1-10.

---

**1** The graphs of $f(x) = 3\cos\left(\frac{x}{2}\right)$ and $g(x) = 3 - \frac{1}{2}x$ are shown below.




How many solutions are there to the equation $3\cos\left(\frac{x}{2}\right) = -\frac{1}{2}x + 3$ for $0 < x < 4\pi$ ?

A. 1

B. 2

C. 3

D. 4

**2** Mr Kim recorded the time it took him to drive to work over five days. The mean time was 37 minutes. The next workday it took Mr Kim 49 minutes to drive to work. What is his new mean time?

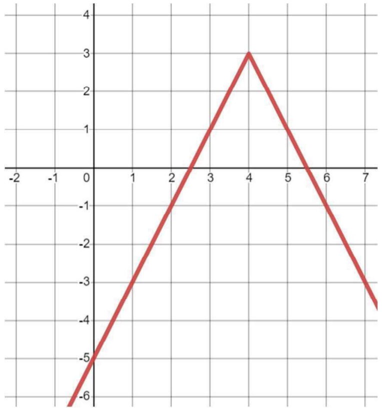
A. 39

B. 43

C. 47

D. 49

**3** What is the equation of the graph shown below?



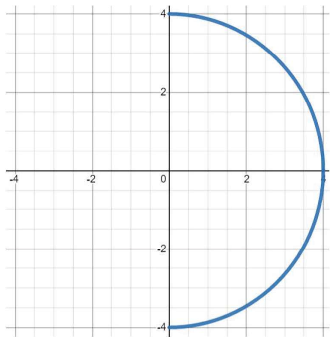
A. $f(x) = -|x - 4| + 3$

B. $f(x) = -2|x + 4| + 3$

C. $f(x) = -|x + 4| + 3$

D. $f(x) = -2|x - 4| + 3$

**4** What type of relationship is represented by the graph shown below?



A. Many-to-many

B. Many-to-one

C. One-to-many

D. One-to-one



**5** An investment of \$7 000 gains 3% per annum for 20 years, with interest calculated monthly.

Which expression gives the value of the investment after 20 years?

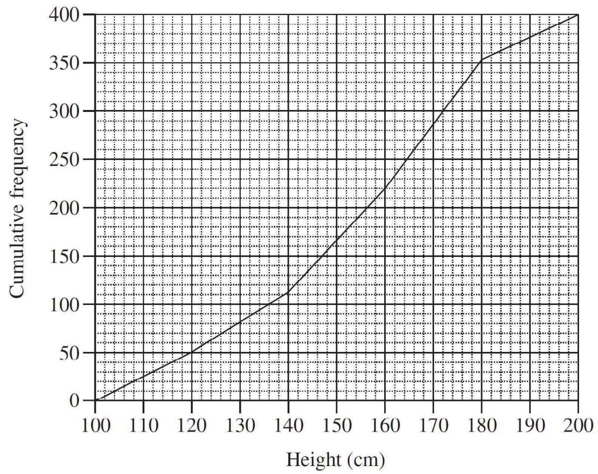
A. $7\,000\,(1 + 0.03)^{20}$

B. $7\,000\left(1 + \frac{0.03}{12}\right)^{240}$

C. $7\,000\,(1 + (0.03 \times 12))^{\frac{20}{12}}$

D. $7000 \times 0.03 \times 20$

**6** The height of 400 students were measured. The results are displayed in the cumulative frequency polygon shown below.




What is the median height of the students?

A. 150

B. 156

C. 165

D. 200

**7** Let $f(x) = \frac{2}{3-x}$ and $g(x) = x - 5$. The domain of $f\big(g(x)\big)$ is

A. All real $x$

B. $x \neq 2$

C. $x \neq 3$

D. $x \neq 8$

**8** The radius and centre of a circle with equation $x^2 + y^2 - 8y - 1 = 0$ is

A. Radius $= 17$ , Centre $(0, -4)$

B. Radius $= 15$ , Centre $(0, 4)$

C. Radius $= \sqrt{15}$ , Centre $(0, -4)$

D. Radius $= \sqrt{17}$ , Centre $(0, 4)$

**9** Find the derivative of $e^{x\cos 3x}$.

A. $e^{3x\sin 3x}$

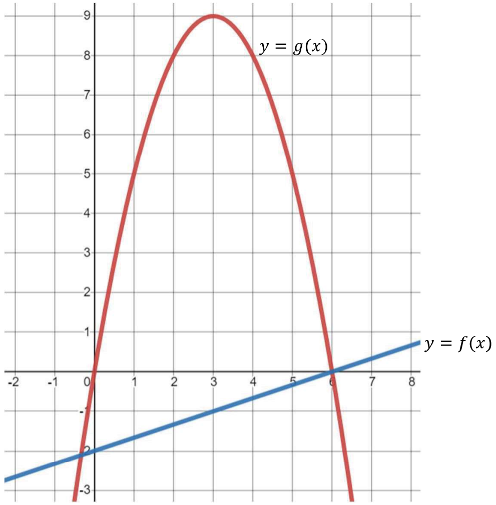
B. $e^{x\cos 3x}$

C. $e^{x\cos 3x}(\cos 3x + 3x\sin 3x)$

D. $e^{x\cos 3x}(\cos 3x - 3x\sin 3x)$



**10** The graphs of $y = g(x)$ and $y = f(x)$ are shown below.




It is known that $\int_0^6 \big(g(x) - f(x)\big)\,dx = 42$.

Find $\int_0^6 g(x)\,dx$

A. 30

B. 31.5

C. 36

D. 38

**Question 11** (3 marks)

Find the equation of the tangent to the curve $y = x^3 - 4x^2$ at the point $(2, -8)$. **3**

**Question 12** (2 marks)

Solve $5 - 3x < 20$, giving your answer in interval notation. **2**



**Question 13** (3 marks)

The function $f(x) = e^x$ is transformed to $g(x) = -e^{3(x+4)}$. **3**

Describe, in words, the sequence of transformations.

**Question 14** (2 marks)

Differentiate $\frac{x^3+2}{x-1}$. **2**

**Question 15** (2 marks)

What is the limiting sum of the following geometric series? **2**

$$-324 + 108 - 36 + 12 \ldots$$

**Question 16** (4 marks)

Solve $2\log_2 x - \log_2(x+6) = 3$. **4**

**Question 17** (3 marks)

Evaluate $\int_0^{\frac{\pi}{2}} \cos\left(\frac{x}{3}\right)\ dx$. **3**



**Question 18** (4 marks)

Mr Tan played a table tennis match. The number of points, $X$, that he scores in each set is a random variable with probability distribution given by:

| $X = x$ | 9 | 10 | 11 | 12 |
|---|---|---|---|---|
| $P(x)$ | 0.2 | $a$ | $b$ | 0.1 |

(a) Evaluate $a$ and $b$ given that Mr Tan has an expected score of 10.4 per set. **3**

(b) Calculate the variance of $X$. **1**

**Question 19** (6 marks)

A semi circle with radius 5 has equation $f(x) = \sqrt{25 - x^2}$.

(a) Use the trapezoidal rule with five function values to approximate the area of the semi circle. Answer correct to 3 decimal places. **3**

(b) Calculate the difference between the actual area of the semi circle and the approximation by the trapezoidal rule. Answer correct to 3 decimal places. **2**

(c) Explain why the trapezoidal rule gives an estimate that is lower than the actual area of the semi circle. **1**



**Question 20** (3 marks)

Mr Huynh recorded how many push ups he did each day for thirty days. **3**

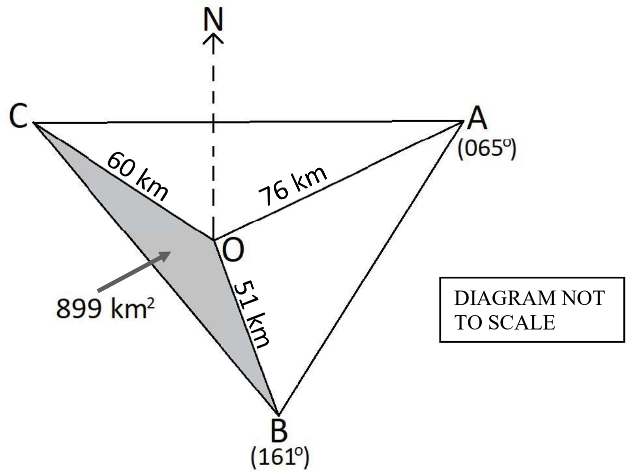
The results are shown in the frequency table.

| **Push ups per day** | **Frequency** |
|---|---|
| 100 | 10 |
| 120 | 6 |
| 140 | 12 |
| 160 | 1 |
| 180 | 0 |
| 200 | 0 |
| 220 | 1 |

Using calculations, decide whether or not 220 push ups was an outlier.

**Question 21** (4 marks)

The diagram shows three triangular blocks of land which all share a common point $O$. The distance and true bearings of the corners of the blocks from point $O$ are shown. **4**




The area of the obtuse-angled triangle $BOC$ is 899 km$^2$.

Calculate the true bearing of the point $C$ from the point $O$, correct to the nearest degree.



**Question 22** (3 marks)

Mrs Briggs notices that the amount of emails she receives per week from students is cyclical. Every 12 weeks she receives the highest number of emails. The average amount of emails per week is 25 and the highest number of emails per week is 40. **3**

The number of emails Mrs Briggs receives each week, $f(t)$, is given by

$$f(t) = a\cos(bt) + c$$

where $t$ is the number of weeks after 1 January 2022 and $0 \leq t \leq 52$.

The number of emails is at a maximum when $t = 0$.

Find the values $a, b$ and $c$.

**Question 23** (4 marks)

Currently, there is a mouse plague in Western NSW. The population of mice can be found using the formula $N = 10(e^{kt})$, where $N$ is the number of mice and $t$ is the time in months. **4**

The plague started with $10$ mice and increased to $100$ mice after $3$ months.

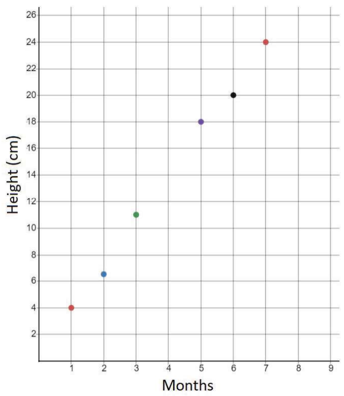
By first finding the value of $k$, show that the exact rate of increase after 12 months is $\frac{10\ln 10}{3}\left(e^{\ln 10000}\right)$ mice/month.

**Question 24** (5 marks)

Mrs Zhou studied the growth of one of her indoor plants over several months.

The collected data is shown in the table and graphed in the scatterplot shown.

| Months since planting, $m$. | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Height of plant, $h$ cm | 4 | 6.5 | 11 | $x$ | 18 | 20 | 24 |




(a) Determine the equation of the least-squares regression line for this data. **1**

(b) Calculate Pearson's correlation coefficient for the data, correct to three decimal places. **1**

**Question 24 continues on page 19**



Question 24 (Continued)

(c) Mrs Zhou missed recording the plants' height at four months. Use the least squares regression line to estimate the height, $x$, of the plant at four months. Answer correct to one decimal place. **1**

(d) Discuss the accuracy of your answer to part (c). **1**

(e) Interpret the value of the gradient of the least squares regression line in the given context. **1**

**Question 25** (3 marks)

Prove that $\tan\theta\sin\theta + \cos\theta = \sec\theta$. **3**

**Question 26** (5 marks)

Sketch the graph of the curve $y = x^3 - 3x^2 + 1$, labelling the stationary points and point of inflection. **5**

Do NOT determine the $x$-intercepts of the curve.



**Question 27** (2 marks)

Mr Pollard is creating a website. Every weekend, apart from the first, he adds the same number of pages to his website. After four weekends he has added 25 pages and after eleven weekends he has added 74 pages.

(a) Show that the total number amount of pages, $P$, added after the $n$th weekend can be modelled by the recurrence relation **1**

$$P_n = P_{n-1} + 7\ , \quad \text{where } n = 2, 3, 4 \ldots$$

(b) Given that on the first weekend he only added 4 pages, use the recurrence relation to find the total number of pages Mr Pollard has added after the third weekend. **1**

**Question 28** (4 marks)

Dr Katyal pours 500 mL of liquid fertiliser into a previously empty bucket. She then fills the bucket with water at a rate of $\frac{dV}{dt}$ litres per minute, where $\frac{dV}{dt} = \frac{5}{t+1}$ and $t$ is time in minutes. **4**

Find the time, to the nearest second, at which there is 10 L of liquid in the bucket.

**Question 29** (7 marks)

Each year, the number of ice creams sold, $c(t)$, can be modelled by the function

$$c(t) = 200\cos\left(\frac{\pi}{26}t\right) + 300$$

where $t$ is the number of weeks after January 1st.

(a) Find how many ice creams were sold on January 1st. **1**

(b) What are the two values of $t$ for which there were 400 ice creams sold? **3**

**Question 29 continues on page 23**



Question 29 (continued)

(b) Sketch the graph of $c(t)$ for $0 \leq t \leq 52$. **3**

**Question 30** (3 marks)

(a) Show that the derivative of $2x\log_5 x$ is $2\log_5 x + \frac{2x}{x\ln 5}$ **2**

(b) Hence, find $\int_0^{\frac{\pi}{2}} \log_5 x + \frac{1}{\ln 5}\, dx$. **1**

**Question 31** (4 marks)

Ms Alrubai and Mrs Norman are employed as doctors. They both start on the same annual salary. However, Ms Alrubai negotiated an annual salary increase of $5 000, while Mrs Norman negotiated an annual salary increase of 7%. **4**

Find their starting annual salary given that after 16 years they have both made the same amount of money in total.



**Question 32** (6 marks)

If Mr Ferrarin wakes up late, the probability that he exercises is $\frac{1}{3}$, and if he wakes up on time the probability that he exercises is $\frac{4}{5}$.

(a) In a particular week, Mr Ferrarin wakes up on time on Thursday morning and late on Friday morning. **2**

Find the probability that Mr Ferrarin exercised on at least one of these days.

(b) The probability that Mr Ferrarin wakes up on time on a Monday is $\frac{1}{4}$.

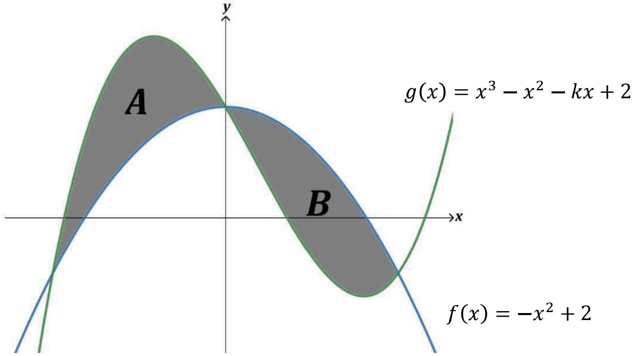
(i) Find the probability that Mr Ferrarin exercises on any given Monday. **2**

(ii) Hence, or otherwise, find the probability that Mr Ferrarin woke up on time on a Monday, given that he exercised that day. **2**

**Question 33** (4 marks) **4**

The graphs of $f(x) = -x^2 + 2$ and $g(x) = x^3 - x^2 - kx + 2$ , where $k$ is a constant, are shown below. **5**

The graphs intersect and create two closed regions, A and B.

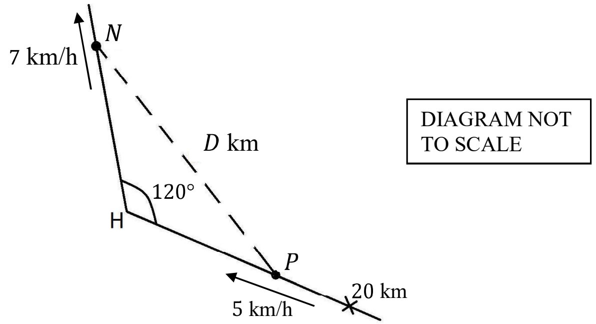



Show that these two regions have the same area.



**Question 34** (5 marks)

Ned Kelly, $N$, left his hideout, $H$, and walked along a straight path at 7 km/h. At the same time that Ned left his hideout, the Police, $P$, are 20 km away from his hideout, walking towards it along a different straight path at 5 km/h. The two paths meet at an angle of 120°.




The distance between Ned Kelly and the police at time $t$ hours is $D$ km.

(a) Show that $D^2 = 39t^2 - 60t + 400$. **2**

(b) Find the minimum distance, to the nearest kilometre, between the police and Ned Kelly. **3**

2022

Penrith High School
HIGHER SCHOOL CERTIFICATE
**TRIAL EXAMINATION**

# Mathematics Advanced

ALTIORA PETO

**Total marks: 100**

**Section I – 10 marks** (pages 2-5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

**Section II – 90 marks** (pages 6-30)

- Attempt Questions 11–34
- Allow about 2 hours and 45 minutes for this section

**NESA NUMBER:** SOLUTIONS

**TEACHER:** ______________________

| | Differentiation | Statistics | Algebra & Graphing | Integration | Series | Trigonometry |
|---|---|---|---|---|---|---|
| **Multiple Choice:** | 9 /1 | 2, 6 /2 | 3, 4, 7, 8 /4 | 10 /1 | 5 /1 | 1 /1 |

| Differentiation | Statistics | Algebra & Graphing | Integration | Logarithms & Exponentials | Probability | Series | Trigonometry |
|---|---|---|---|---|---|---|---|
| /20 | /10 | /9 | /15 | /8 | /10 | /9 | /20 |
| | | | | | | **Total:** | /101 |

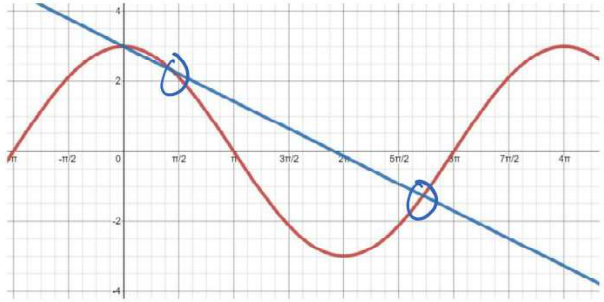
## Section I

**10 marks**
**Attempt questions 1-10**
**Allow about 15 minutes for this section**

Use the multiple-choice answer sheet for Questions 1-10.

---

**1** The graphs of $f(x) = 3\cos\left(\frac{x}{2}\right)$ and $g(x) = 3 - \frac{1}{2}x$ are shown below.



How many solutions are there to the equation $3\cos\left(\frac{x}{2}\right) = -\frac{1}{2}x + 3$ for $0 < x < 4\pi$ ?

don't include $x = 0$

A. 1

B. 2

C. 3

D. 4

**2** Mr Kim recorded the time it took him to drive to work over five days. The mean time was 37 minutes. The next workday it took Mr Kim 49 minutes to drive to work. What is his new mean time?

A. 39

B. 43

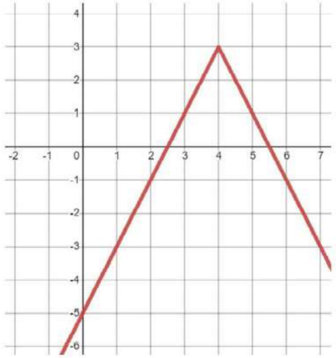
C. 47

D. 49

$$\frac{37 \times 5 + 49}{6}$$

$$= 39$$

**3** What is the equation of the graph shown below?



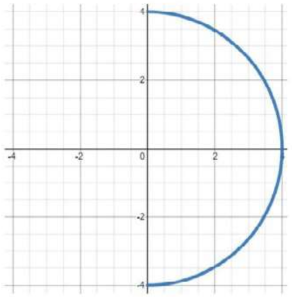
A. $f(x) = -|x - 4| + 3$

B. $f(x) = -2|x + 4| + 3$

C. $f(x) = -|x + 4| + 3$

D. $f(x) = -2|x - 4| + 3$

**4** What type of relationship is represented by the graph shown below?



A. Many-to-Many

B. Many-to-one

C. One-to-many

D. One-to-one

$\frac{0.03}{12}$ $20 \times 12 = 240$

**5** An investment of \$7 000 gains 3% per annum for 20 years, with interest calculated monthly.

Which expression gives the value of the investment after 20 years?

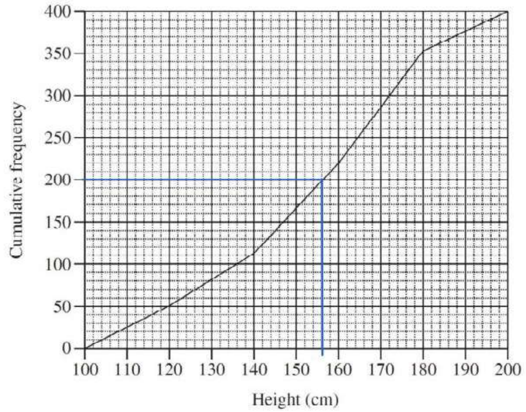
A. $7\,000\,(1 + 0.03)^{20}$

B. $7\,000\left(1 + \frac{0.03}{12}\right)^{240}$

C. $7\,000\,(1 + (0.03 \times 12))^{\frac{20}{12}}$

D. $7000 \times 0.03 \times 20$

**6** The height of 400 students were measured. The results are displayed in the cumulative frequency polygon shown below.




What is the median height of the students?

A. 150

B. 156

C. 165

D. 200

7 Let $f(x) = \frac{2}{3-x}$ and $g(x) = x - 5$. The domain of $f\big(g(x)\big)$ is

A. All real $x$

B. $x \neq 2$

C. $x \neq 3$

D. $x \neq 8$

$\frac{2}{3-(x-5)}$

$= \frac{2}{3-x+5}$

$= \frac{2}{-x+8}$

$\therefore -x+8 \neq 0$

$x \neq 8$

8 The radius and centre of a circle with equation $x^2 + y^2 - 8y - 1 = 0$ is

A. Radius $= 17$, Centre $(0, -4)$

B. Radius $= 15$, Centre $(0, 4)$

C. Radius $= \sqrt{15}$, Centre $(0, -4)$

D. Radius $= \sqrt{17}$, Centre $(0, 4)$

$x^2 + y^2 - 8y + \left(\frac{8}{2}\right)^2 = \left(\frac{8}{2}\right)^2 + 1$

$x^2 + (y-4)^2 = 17$

9 Find the derivative of $e^{x\cos 3x}$.

A. $e^{3x\sin 3x}$

B. $e^{x\cos 3x}$

C. $e^{x\cos 3x}(\cos 3x + 3x\sin 3x)$

D. $e^{x\cos 3x}(\cos 3x - 3x\sin 3x)$

$y = e^{f(x)}$

$y' = f'(x)e^{f(x)}$

$u = x$

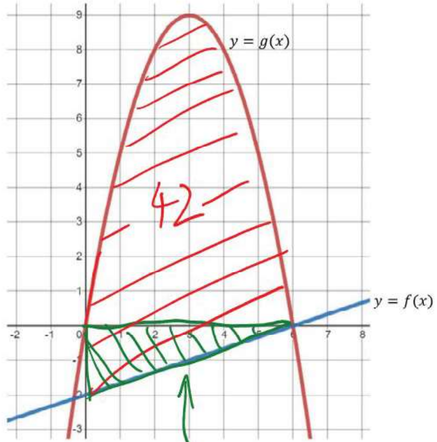
$u' = 1$

$v = \cos 3x$

$v' = -3\sin 3x$

$f'(x) = \cos 3x - 3x\sin 3x$

10 The graphs of $y = g(x)$ and $y = f(x)$ are shown below.




$A = \frac{1}{2} \times 6 \times 2$

$= 6$

It is known that $\int_0^6 (\,g(x) - f(x)\,)dx = 42$.

Find $\int_0^6 g(x)\,dx$.

A. 30

B. 31.5

C. 36

D. 38

$\int_0^6 g(x)\,dx = 42 - 6$

$= 36$

**Question 11** (3 marks)

Find the equation of the tangent to the curve $y = x^3 - 4x^2$ at the point $(2, -8)$. 3

$y - y_1 = m(x - x_1)$

$y' = 3x^2 - 8x$

at $x = 2$

$m = 3(2)^2 - 8(2)$

$= -4$ 1

$y - (-8) = -4(x - 2)$ 1

$y + 8 = -4x + 8$

$y = -4x$ 1

**Question 12** (2 marks)

Solve $5 - 3x < 20$, giving your answer in interval notation. 2

$-3x < 15$

$x > -5$ 1

$(-5, \infty)$ 1

**Question 13** (3 marks)

ALTERNATE:

The function $f(x) = e^x$ is transformed to $g(x) = -e^{3(x+4)}$. $= -e^{3x+12}$ 3

Describe, in words, the sequence of transformations.

Reflection across the $x$-axis
Horizontal dilation by a factor of 3
Horizontal translation, left 4 units

or

Reflection across $x$-axis
Translate left 12 units
Dilate horizontally by a factor of $\frac{1}{3}$

NOTE: The reflection can be placed anywhere, as long as the horizontal transformations are in the correct order.

**Question 14** (2 marks)

Differentiate $\frac{x^3+2}{x-1}$. 2

$u = x^3 + 2$

$u' = 3x^2$

$v = x - 1$

$v' = 1$

$\frac{(3x^2)(x-1) - (x^3+2)(1)}{(x-1)^2}$ 1

$= \frac{3x^3 - 3x^2 - x^3 - 2}{(x-1)^2}$ 1 $= \frac{2x^3 - 3x^2 - 2}{(x-1)^2}$

**Question 15** (2 marks)

What is the limiting sum of the following geometric series?

$-324 + 108 - 36 + 12 \ldots$ 2

$S = \frac{a}{1-r}$

$a = -324$

$r = \frac{108}{-324} = -\frac{1}{3}$ 1

$= \frac{-324}{1-(-\frac{1}{3})}$

$= -243$ 1

**Question 16** (4 marks)

Solve $2\log_2 x - \log_2(x+6) = 3$. **4**

$\log_2 x^2 - \log_2(x+6) = 3$

$\log_2 \frac{x^2}{x+6} = 3$ 1

$\frac{x^2}{x+6} = 2^3$ 1

$x^2 = 8(x+6)$

$x^2 = 8x + 48$

$x^2 - 8x - 48 = 0$

$(x-12)(x+4) = 0$

$x = 12$ or $-4$ 1

$x = 12 \quad (x \geq 0)$ 1

* students need to revise the log laws.

$x = \frac{-1 \pm \sqrt{57}}{2}$ ↳ 2mks

*most students forgot the last step.

**Question 17** (3 marks)

Evaluate $\int_0^{\frac{\pi}{2}} \cos\left(\frac{x}{3}\right)\, dx$. **3**

$= \int_0^{\frac{\pi}{2}} \cos\left(\frac{1}{3}x\right) dx$ $\quad f(x) = \frac{1}{3}x$, $f'(x) = \frac{1}{3}$

$= 3\int_0^{\frac{\pi}{2}} \frac{1}{3}\cos\left(\frac{1}{3}x\right) dx$

$I = \left[3\sin\frac{x}{3}\right]_0^{\pi/2}$

$= 3\left[\sin\left(\frac{1}{3}x\right)\right]_0^{\frac{\pi}{2}}$ 1

$= 3\left[\sin\left(\frac{1}{3}\times\frac{\pi}{2}\right) - \sin\left(\frac{1}{3}\times 0\right)\right]$ 1

$= 3\left[\sin\left(\frac{\pi}{6}\right) - \sin 0\right]$

$3\left(\frac{1}{2} - 0\right)$

$= \frac{3}{2}$ 1

**Question 18** (4 marks)

Mr Tan played a table tennis match. The number of points, $X$, that he scores in each set is a random variable with probability distribution given by:

| $X = x$ | 9 | 10 | 11 | 12 |
|---|---|---|---|---|
| $P(x)$ | 0.2 | $a$ | $b$ | 0.1 |

(a) Evaluate $a$ and $b$ given that Mr Tan has an expected score of 10.4 per set. **3**

$0.2 + a + b + 0.1 = 1$

$a + b = 0.7$

$a = 0.7 - b$ 1

$9\times0.2 + 10a + 11b + 12\times0.1 = 10.4$ 1

$1.8 + 10a + 11b + 1.2 = 10.4$

$10a + 11b = 7.4$

$\therefore 10(0.7-b) + 11b = 7.4$

$7 - 10b + 11b = 7.4$

$b = 0.4$

$a = 0.7 - 0.4$

$= 0.3$ 1

$a = 0.3,\ b = 0.4$

If you just had $a = 0.3$ and $b = 0.4$ with no working ∴ max. 1 mark.

(b) Calculate the variance of $X$. **1**

| $x^2$ | 81 | 100 | 121 | 144 |
|---|---|---|---|---|

right or wrong.

$E(x^2) = 81\times0.2 + 100\times0.3 + 121\times0.4 + 144\times0.1$

$= 109$

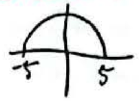
$E(x^2) - \mu^2 = 109 - 10.4^2 = 109 - 108.16$

$= 0.84$

**Question 19** (6 marks)

means 4 intervals.

A semi-circle with radius 5 has equation $f(x) = \sqrt{25 - x^2}$.




(a) Use trapezoidal rule with five function values to approximate the area of the semi-circle. Answer correct to 3 decimal places. **3**

$$A = \frac{b-a}{2n}\{f(a)+f(b)+2[f(x_1)+f(x_2)+f(x_3)]\}$$

$a=-5 \quad f(-5)=0$

$x_1=-2.5 \quad f(-2.5)=\sqrt{18.75}$

$x_2=0 \quad f(0)=5$

$x_3=2.5 \quad f(2.5)=\sqrt{18.75}$

$b=5 \quad f(5)=0$

$$= \frac{5-(-5)}{2\times 4}\{0+0+2[\sqrt{18.75}+5+\sqrt{18.75}]\}$$

$$= 34.15063509$$

$$= 34.151 \text{ units}^2 \ (3dp)$$

1 1 1

*very badly done.
* You must write out the formula and line of substitution

(b) Calculate the difference between the actual area of the semi-circle and the approximation by trapezoidal rule. Answer correct to 3 decimal places. **2**

$A = \frac{1}{2}\pi\times 5^2$

$= \frac{25}{2}\pi$

$= 39.2699$

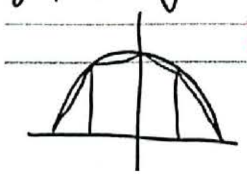
$\text{Diff} = \frac{25}{2}\pi - 34.151$

$= 5.119 \text{ units}^2$ less by trapezoidal rule.

1 1

(c) Explain why trapezoidal rule gives an estimate that is lower than the actual area of the semi-circle. **1**

Trapezoidal rule gives a lower estimate because when you draw the trapeziums on a concave down graph, they will always be below the curve, as shown in the diagram.





**Question 20** (3 marks)

33

Mr Huynh recorded how many push ups he did each day for thirty days.

The results are shown in the frequency table.

| Push ups per day | Frequency | c.f. |
|---|---|---|
| 100 | 10 | 10 |
| 120 | 6 | 16 |
| 140 | 12 | 28 |
| 160 | 1 | 29 |
| 180 | 0 | 29 |
| 200 | 0 | 29 |
| 220 | 1 | 30 |

Using calculations, decide whether or not 220 push ups was an outlier.

Median = 15.5th score = 120

$Q_1$ = 8th score = 100

$Q_3$ = 23rd score = 140

IQR = 140 − 100 = 40

1

$Q_3 + 1.5\times IQR = \cancel{100}\ 140 + 1.5\times 40$

$= \cancel{160}\ 200$

1

$220 > \cancel{160}\ 200$

∴ It is an outlier.

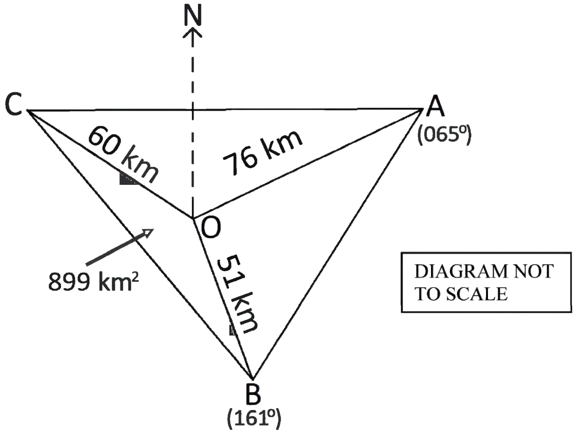
1

OC

Answer only = 1 mark.

**Question 21** (4 marks)

The diagram shows three triangular blocks of land which all share a common point $O$. The distance and true bearings of the corners of the blocks from point $O$ are shown. **4**




The area of the obtuse-angled triangle $BOC$ is 899 km².

Calculate the true bearing of the point $C$ from the point $O$, correct to the nearest degree.

Bearing $= 161 + \angle BOC$

$= 161 + 144$

$= 305°$ — 1

CFM

$A = \frac{1}{2}ab\sin C$

$899 = \frac{1}{2} \times 51 \times 60 \times \sin(\angle BOC)$ — 1

$\frac{899}{1530} = \sin(\angle BOC)$

$\angle BOC = \sin^{-1}\left(\frac{899}{1530}\right)$

$= 35.99$

$= 36°$ — 1

But it is an obtuse angled triangle.

$\therefore 180 - 36 = 144$ — 1

Many students ignored the "obtuse-angled" part of the question or didn't know how to find the obtuse angle (adding 90° was a common error).

Some ignored the area of the triangle part of the question. Instead using cosine or sine rule to find irrelevant sides or angles.

There are no tricks in the HSC. If the information is there then use it!

Also use the reference sheet!



**Question 22** (3 marks)

Mrs Briggs notices that the amount of emails she receives per week from students is cyclical. Every 12 weeks she receives the highest number of emails. The average amount of emails per week is 25 and the highest number of emails per week is 40. **3**

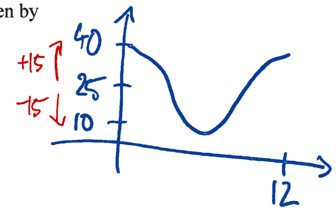
The number of emails Mrs Briggs receives each week, $f(t)$, is given by

$$f(t) = a\cos(bt) + c$$

Where $t$ is time in weeks and $0 \leq t \leq 52$.

The number of emails is at a maximum when $t = 0$.

Find the values $a, b$ and $c$.




period $= \frac{2\pi}{b}$

$12 = \frac{2\pi}{b}$

$b = \pi/6$

$\boxed{a = 15 \quad b = \pi/6 \quad c = 25}$

Draw a picture! It's easier.

After feedback from last exam, many students learnt the period formula. However some have still not.

**Question 23** (4 marks)

Currently, there is a mouse plague in Western NSW. The population of mice can be found using the formula $N = 10(e^{kt})$, where $N$ is the number of mice and $t$ is the time in months. **4**

The plague started with 10 mice and increased to 100 mice after 3 months.

By first finding the value of $k$, show that the exact rate of increase after 12 months is $\frac{10\ln 10}{3}\left(e^{\ln 10000}\right)$ mice/month.

Rate = derivative.

$100 = 10(e^{3k})$

$10 = e^{3k}$

$\ln 10 = 3k$

$k = \frac{\ln 10}{3}$ — 1

$\therefore N = 10e^{\frac{\ln 10}{3}t}$

Always leave as exact value. Never decimals, unless the Q asks

$N' = 10 \times \frac{\ln 10}{3} e^{\frac{\ln 10}{3}t}$ — 1

at $t = 12$

$N' = \frac{10\ln 10}{3} e^{\frac{\ln 10}{3} \times 12}$ — 1 mark for substituting $t = 12$ into any equation

$= \frac{10\ln 10}{3} e^{4\ln 10}$

$= \frac{10\ln 10}{3} e^{\ln 10^4}$

$= \frac{10\ln 10}{3} e^{\ln 10000}$

Must have both of these lines for final mark. This is a show Q. You can't jump from $4\ln 10$ to $\ln 10000$

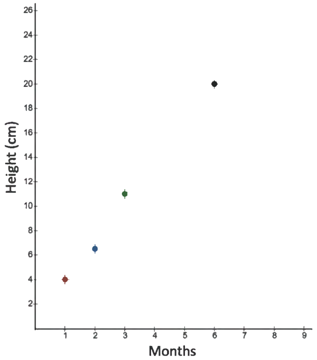
This was poorly done. It is a very common exam question. If you didn't get it then please practice this type of question.

**Question 24** (5 marks)

Mrs Zhou studied the growth of one of her indoor plants over several months.

The collected data is shown in the table and graphed in the scatterplot shown.

| Months since planting, $m$. | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Height of plant, $h$ cm | 4 | 6.5 | 11 | $x$ | 18 | 20 | 24 |




(a) Determine the equation of the least-squares regression line for this data. **1**

$A = 0.488 \quad B = 3.357$

$h = 3.357m + 0.488$

(b) Calculate Pearson's correlation coefficient for the data, correct to three decimal places. **1**

$r = 0.997$

Overall, all of Q24 was well done. Again a big improvement from last exam. Again these are very common (and very easy) questions. If you still didn't get it then practice these types of questions!

**Question 24 continues on page 19**



Question 24 (Continued)

(c) Mrs Zhou missed recording the plants' height at four months. Use the least squares regression line to estimate the height, $x$, of the plant at four months. Answer correct to one decimal place. **1**

$h = 3.357(4) + 0.488$

$= 13.9$ cm (1dp)

Show the substitution! Otherwise you may not get a CFM if your part a was wrong.

(d) Discuss on the reliability of your answer to part (c). **1**

The estimate is very reliable, as Pearsons correlation Coefficient is 0.997 and it is interpolation.

Interpolation on its own does not mean a prediction is reliable, it needs to be combined with a Pearsons' Correlation Coefficient close to 1 or -1. Just as extrapolation on its own does not mean a prediction is unreliable.

(e) Interpret the value of the gradient of the least squares regression line in the given context. **1**

The gradient means the plant grows, on average, 3.357 cm per month.

Key words in the question are Interpret, value of the gradient, context

You must mention the actual value (3.357) and what it means (growth in cm/month)

Gradient = rate = $\frac{\text{rise}}{\text{run}}$ in this context $\frac{\text{cm}}{\text{month}}$

**Question 25** (3 marks)

Prove that $tan\theta sin\theta + cos\theta = sec\theta$. **3**

Again this feedback was given in a past exam and still some didn't learn it.

LHS $= \tan\theta\sin\theta + \cos\theta$

$= \frac{\sin\theta}{\cos\theta}\sin\theta + \cos\theta$

$= \frac{\sin^2\theta}{\cos\theta} + \cos\theta$

(1) either line

$= \frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta}$

(1) either line

This is a show question. Don't skip steps! Many skipped the first line. Lucky to get the mark.

$= \frac{1}{\cos\theta}$

$= \sec\theta$

BOTH (1)

$=$ RHS

The verb is sketch. Your graph is the important bit. Take your time and don't make it small. Use another page if needed.

**Question 26** (5 marks)

Sketch the graph of the curve $y = x^3 - 3x^2 + 1$, labelling the stationary points and point of inflection. 5

Do NOT determine the $x$-intercepts of the curve.

Label coordinates for 1 mark.

y-int = 1

stpt when $y' = 0$

$y' = 3x^2 - 6x$

$0 = 3x(x-2)$

$x = 0$ or $2$

$y = (0)^3 - 3(0)^2 + 1 = 1$ $\quad$ $y = (2)^3 - 3(2)^2 + 1 = -3$

possible pt of inflexion when $y'' = 0$

$y'' = 6x - 6$

$0 = 6x - 6$

$x = 1$

$y = (1)^3 - 3(1)^2 + 1 = -1$

TEST:

| $x$ | $-0.1$ | $0$ | $0.1$ |
|---|---|---|---|
| $y'$ | $0.63$ | $0$ | $-0.57$ |
| | / | — | \ |

$(0,1)$ is max

| $x$ | $1.9$ | $2$ | $2.1$ |
|---|---|---|---|
| $y'$ | $-0.57$ | $0$ | $0.63$ |
| | \ | — | / |

$(2,-3)$ is min

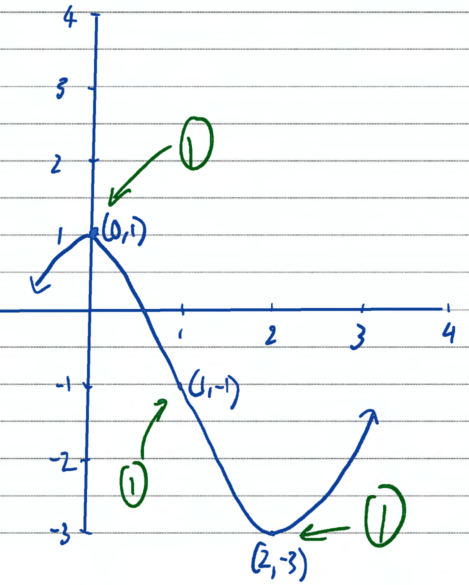
TEST:

| $x$ | $0.9$ | $1$ | $1.1$ |
|---|---|---|---|
| $y''$ | $-0.6$ | $0$ | $0.6$ |
| | ∩ | | ∪ |

$\therefore (1,-1)$ is pt of inflection

Many students didn't test the point of inflection.

Some students lost a mark for really bad scales. There are lines on the page, use them for your scale! They also make bad scales obvious.






**Question 27** (2 marks)

Mr Pollard is creating a website. Every weekend, apart from the first, he adds the same number of pages to his website. After four weekends he has added 25 pages and after eleven weekends he has added 74 pages.

(a) Show that the total number amount of pages, $P$, added after the $n$th weekend can be modelled by the recurrence relation 1

$$P_n = P_{n-1} + 7, \quad \text{where } n = 2, 3, 4 \ldots$$

$P_4 + 7d = P_{11}$

$74 + 7d = 25$

$7d = 49$

$d = 7$

$\therefore P_n = P_{n-1} + 7$

(b) Given that on the first weekend he only added 4 pages, use the recurrence relation to find the total number of pages Mr Pollard has added after the third weekend. 1

Must have this! (use the recurrence relation.)

$P_0 = 4$

$P_1 = 4 + 7 = 11$

$P_2 = 11 + 7 = 18$

$\therefore$ He has 18 pages after the third weekend.

**Question 28** (4 marks)

Dr Katyal pours 500 mL of liquid fertiliser into a previously empty bucket. She then fills the bucket with water at a rate of $\frac{dV}{dt}$ litres per minute, where $\frac{dV}{dt} = \frac{5}{t+1}$ and $t$ is time in minutes. 4

Find the time, to the nearest second, at which there is 10 L of liquid in the bucket.

$\int \frac{dV}{dt}\, dt = V$

$\boxed{\int \frac{f'(x)}{f(x)} = \ln|f(x)| + C \quad f(x) = t+1 \quad f'(x) = 1}$

This formula is on the reference sheet!

$V = \int \frac{5}{t+1}\, dt$

$= 5 \int \frac{1}{t+1}\, dt$

$= 5\ln|t+1| + C$ — 1

when $t = 0$ $V = 0.5$

$0.5 = 5\ln|0+1| + C$

$C = 0.5$ — 1

$V = 5\ln|t+1| + 0.5$

when $V = 10$

$10 = 5\ln|t+1| + 0.5$

$\frac{9.5}{5} = \ln|t+1|$

$t + 1 = e^{\frac{9.5}{5}}$ — 1

$t = e^{\frac{9.5}{5}} - 1$

$= 5.686$

$= 5$ min $41$ sec. — 1

This is a very common exam question. If you got it wrong then practice!

Some students wrote 6 sec, but $t$ is in minutes.

**Question 29** (7 marks)

Each year, the number of ice creams sold, $c(t)$, can be modelled by the function

$$c(t) = 200\cos\left(\frac{\pi}{26}t\right) + 300$$

where $t$ is the number of weeks after January 1st.

(a) Find how many ice creams were sold on January 1st. **1**

$c(0) = 200\cos\left(\frac{\pi}{26}\times 0\right) + 300$

$= 500$ 1

(b) What are the two values of $t$ for which there were 400 ice creams sold? **3**

$400 = 200\cos\left(\frac{\pi}{26}t\right) + 300$

$100 = 200\cos\left(\frac{\pi}{26}t\right)$

$\frac{1}{2} = \cos\left(\frac{\pi}{26}t\right)$ 1

Ref $\angle = \frac{\pi}{3}$ S A T C

$\frac{\pi}{26}t = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$ 1

$= \frac{\pi}{3}, \frac{5\pi}{3}$

$t = \frac{26}{3}, \frac{130}{3}$ 1

* Most students did it well.

* If only one solution 2/3

* If degrees then final answer incorrect

↳ unless converted to radians again.

Question 29 continues on page 23

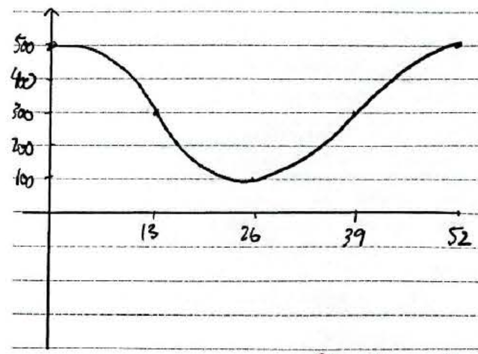
Question 29 (continued)

(b) Sketch the graph of $c(t)$ for $0 \le t \le 52$. **3**

period $= \frac{2\pi}{a}$

$= \frac{2\pi}{\pi/26}$

$= 52$




1 shape

1 Amplitude from 500 to 100

1 Period (and correctly labelled axis)

* If arrows then incorrect

* If not symmetrical then -1.

Question 30 (3 marks)

(a) Show that the derivative of $2x\log_5 x$ is $2\log_5 x + \frac{2x}{x\ln 5}$ 2

Need to show this

1

$u = 2x$

$u' = 2$

$v = \log_5 x$

$v' = \frac{1}{(\ln 5)x} = \frac{1}{x\ln 5}$

$2\log_5 x + 2x\left(\frac{1}{x\ln 5}\right)$ 1

$= 2\log_5 x + \frac{2x}{x\ln 5}$

* Need to show line of substitution otherwise copying from answers.

(b) Hence, find $\int_0^{\pi/2} (\log_5 x + \frac{1}{\ln 5})\, dx$. 1

$2\log_5 x + \frac{2x}{x\ln 5}$

$= 2\log_5 x + \frac{2}{\ln 5}$

$= 2(\log_5 x + \frac{1}{\ln 5})$

$\frac{1}{2}\int 2(\log_5 x + \frac{1}{\ln 5})dx$

$= \left[\frac{1}{2}(2x\log_5 x)\right]_0^{\pi/2}$

$= \left[x\log_5 x\right]_0^{\pi/2}$ = undefined as $\log_5(0)$ is undefined

Question 31 (4 marks)

Ms Alrubai and Mrs Norman are employed as doctors. They both start on the same annual salary. However, Ms Alrubai negotiated an annual salary increase of \$5 000, while Mrs Norman negotiated an annual salary increase of 7%. 4

Find their starting annual salary given that after 16 years they have both made the same amount of money in total.

Alrubai - AP

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{16} = \frac{16}{2}[2a + (16-1)5000]$

$= 8[2a + 75000]$

$= 16a + 600\,000$ 1

Norman - GP

$S_{16} = \frac{a(1.07^{16}-1)}{1.07-1}$

$= \frac{a(1.07^{16}-1)}{0.07}$

$= \left(\frac{1.07^{16}-1}{0.07}\right)a$ 1

$\therefore 16a + 600\,000 = \left(\frac{1.07^{16}-1}{0.07}\right)a$ 1

$16a - \left(\frac{1.07^{16}-1}{0.07}\right)a = -600\,000$

$\left(16 - \left(\frac{1.07^{16}-1}{0.07}\right)\right)a = -600\,000$

$a = \frac{-600\,000}{\left(16 - \left(\frac{1.07^{16}-1}{0.07}\right)\right)} = \$50\,470.84$ 1

* Most students used term formula instead of sum ∴ -2 as it makes it easier

* If term formula used incorrectly, -1.

Question 32 (6 marks)

If Mr Ferrarin wakes up late, the probability that he exercises is $\frac{1}{3}$, and if he wakes up on time the probability that he exercises is $\frac{4}{5}$.

(a) In a particular week, Mr Ferrarin wakes up on time on Thursday morning and late on Friday morning. 2

Find the probability that Mr Ferrarin exercised on at least one of these days.

1 - P(doesn't work out)

$= 1 - \left(\frac{1}{5} \times \frac{2}{3}\right)$ 1

$= \frac{13}{15}$ 1

(b) The probability that Mr Ferrarin wakes up on time on a Monday is $\frac{1}{4}$.

(i) Find the probability that Mr Ferrarin exercises on any given Monday. 2

$\frac{1}{4} \times \frac{4}{5} + \frac{3}{4} \times \frac{1}{3}$ 1

$= 0.45$ 1

(ii) Hence, or otherwise, find the probability that Mr Ferrarin woke up on time on a Monday, given that he exercised that day. 2

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{\frac{1}{4} \times \frac{4}{5}}{0.45}$ 1 $= \frac{4}{9}$ 1

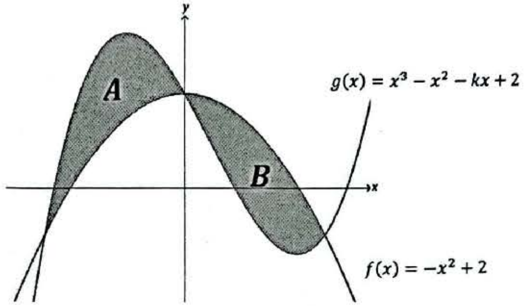


Problem with these questions: If one mistake is made, hard to do CFG. Most mistakes made question easier ∴ can't get 1/2

**Question 33** (4 marks) 4

The graphs of $f(x) = -x^2 + 2$ and $g(x) = x^3 - x^2 - kx + 2$, where $k$ is a constant, are shown below. 5

The graphs intersect and create two closed regions, A and B.




Show that these two regions have the same area.

pts of intersection → $x^3 - x^2 - kx + 2 = -x^2 + 2$

$x^3 - kx = 0$

$x(x^2 - k) = 0$

$x = 0$ or $x^2 - k = 0$

$x = \pm\sqrt{k}$ ①

For A → $(x^3 - x^2 - kx + 2) - (-x^2 + 2)$
$= x^3 - kx$

$A = \int_{-\sqrt{k}}^{0} x^3 - kx \, dx$

$= \left[\frac{x^4}{4} - \frac{kx^2}{2}\right]_{-\sqrt{k}}^{0}$

$= (0) - \left(\frac{(-\sqrt{k})^4}{4} - \frac{k(\sqrt{k})^2}{2}\right)$

$= -\frac{k^2}{4} + \frac{k^2}{2}$

$= +\frac{k^2}{4}$ ①

For B → $(-x^2 + 2) - (x^3 - x^2 - kx + 2)$
$= x^3 + kx$

$B = \int_{0}^{\sqrt{k}} x^3 + kx$

$= \left[\frac{x^4}{4} + \frac{kx^2}{2}\right]_{0}^{\sqrt{k}}$

$= \left(\frac{(\sqrt{k})^4}{4} + \frac{k(\sqrt{k})^2}{2}\right) - (0)$

$= -\frac{k^2}{4} + \frac{k^2}{2}$

$= +\frac{k^2}{4}$ ①

$\therefore A = B$ ①

* Most students over complicated question.

* If indefinite integral then −1 because C, needs to equal C.

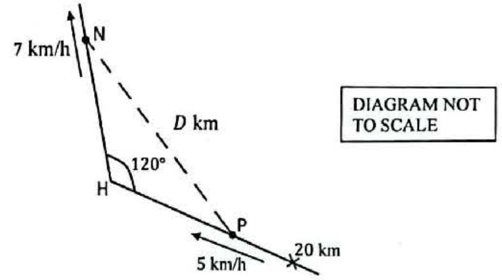
* If incorrect intersection is found then CFE.

* If assumption is made about odd functions without



**Question 34** (5 marks)

Ned Kelly, $N$, left his hideout, $H$, and walked along a straight path at 7 km/h. At the same time that Ned left his hideout, the Police, $P$, are 20 km away from his hideout, walking towards it along a different straight path at 5 km/h. The two paths meet at an angle of 120°.




The distance between Ned Kelly and the police at time $t$ hours is $D$ km.

(a) Show that $D^2 = 39t^2 - 60t + 400$. 2

$P = 20 - 5t$ $\quad N = 7t$ $\quad$ * Done poorly.

$c^2 = a^2 + b^2 - 2ab\cos C$

$D^2 = (20-5t)^2 + (7t)^2 - 2(20-5t)(7t)\cos(120)$ ①

$= 400 - 200t + 25t^2 + 49t^2 - (280t - 70t^2)(-\frac{1}{2})$

$= 400 - 200t + 74t^2 + 140t - 35t^2$

$= 39t^2 - 60t + 400$ ①

(b) Find the minimum distance, to the nearest kilometre, between the police and Ned Kelly. 3

D is a minimum when $D^2$ is.

$\therefore$ Find min st pt for $D^2$

$\frac{dD^2}{dt} = 78t - 60$

st pt when $\frac{dD^2}{dt} = 0$

$0 = 78t - 60$

$t = \frac{60}{78} = \frac{10}{13}$ ①

$D^2 = 39\left(\frac{10}{13}\right)^2 - 60\left(\frac{10}{13}\right) + 400$

$= 371.5976331$

① $D = 19.28$ km (2dp)

①

| $t$ | 0.7 | $\frac{10}{13}$ | 0.8 |
|---|---|---|---|
| $\frac{dD^2}{dt}$ | −5.4 | 0 | 2.4 |
| | \ | — | / |

$\therefore$ min st pt.

* Most students forgot to test to show minimum
OR
use property of original function (concave up parabola).